Journal of Basic and Applied Engineering Research
p-ISSN: 2350-0077; e-ISSN: 2350-0255; Volume 5, Issue 3; January-March, 2018, pp. 231-235

http://www.krishisanskriti.org/Publication.html

# Comparative Analysis for a Simpler System Design Using Model Order Reduction Techniques

**Rajat Vats and S.K. Jha**

*[1,2]Division of ICE, NSIT, New Delhi, India*
*E-mails: [1]rajat.vats93@gmail.com, [2]jhask271@gmail.com*

**Abstract—***This paper mainly focuses on the realization of a reduced order model substitute to a higher order system based on proposed method of Aggregation which harnesses the continued fraction technique. In order to investigate the efficacy of the proposed technique, it is compared with the V Krishnamurthy's approach on reduced order modeling by taking a fourth order system. Subsequently this fourth order system is reduced to second order system using both the methods. The various performance parameters of the reduced systems and the original system obtained from the graphs are compared and analyzed. It is observed that the performance parameters of the proposed method are more close to original system than by V Krishnamurthy's approach.*

**Keywords***: Model Order Reduction; Aggregation by continued fraction; V Krishnamurthy's approach.*

## 1. INTRODUCTION

Design and development of model order reduction techniques has always remained an inquisitive topic to control engineers for decades largely due to its physical simplicity. The model order reduction problem may be defined as the problem of finding an easier mathematical model for a complex system. The basic philosophy is to preserve the important dynamic characteristics of the process, while certain less important characteristics are ignored and complexities are eliminated. Thus, the model order reduction can be achieved by at wo-step process. First step is to identify the dominant and non-dominant subsystem of the higher order system and second step being the elimination of the non-dominant subsystem.

This problem eluded the scientific world for long time especially when it warranted the design of low order controllers for high order plants. The advanced controller design method tends to supply controllers with order comparable to the plant order and therefore controllers are often of high order. But practical implementation of such a high order controller is not an easy task. Intuitive understanding of how the controller is functioning and actual implementation in a reliable manner are major tasks with a high order controller which necessitated for techniques of controller order reduction. But in order to save hardware resources and avoiding numerical difficulties, the design of higher order controller is avoided. It is found that problems of controller reduction are more difficult than those of model reduction.

Modern technology is confronted with large scale systems which are large in dimension and stochastic by nature which is too complex to be studied without model order reduction. The dynamics of physical systems are generally described in terms of number of simultaneous linear differential equation with constant coefficients or in turn in the state variable form [1-3]. But for many processes the order of the system matrix may be too large to be worked in original form. In such conditions it is customary to study the process by approximating it to a simpler reduced model.

The higher order models are generally complex. It takes lots of time to evaluate. Many situations we come across in daily life can be modeled by higher order transfer functions. But the evaluation of these higher order systems requires immense calculations. We can signify these higher order systems by subsequent lower order systems, such that the original system and the approximated models have almost identical characteristics and responses. The mathematical models of real-life processes create challenges when used in numerical simulations, due to large size (dimension) and complexity they have. Model order reduction lowers computational complexity of higher order systems.

Many theories has been given to accomplish the purpose by estimating the dominant part of the large system and finding a simpler reduced form of system representation that has its behaviour akin to the original system. Hence model reduction has become a ubiquitous tool in analysis and simulation of dynamical system. Reduced order modelling thus is an important issue rather it may be used as a tool in the coming days in control systems. In literature, various research articles have been published for reducing the order of linear systems. Time Domain Effects of Model Order Reduction has been presented in [4]. A unified derivation and critical review of model approaches to model reduction [5] and similarly the cost decomposition of linear systems with application to model reduction is described in [6].

Many modifications to the existing methods are done and presented. Dynamics separation of induction machine models and Reduction of the model order of permanent magnet synchronous machine are done to get a reduced order system which closely matches the corresponding higher order system[7-8].Different comparative studies and analysis is developed between the existing techniques of model order reduction [9-23].

In this paper we have compared two different methods of model order reduction which uses Aggregation by continued fraction method and V Krishnamurthy's approach. The approaches are analyzed on the basis of Rise Time, Settling time, Peak time, Peak Value by considering an example where a higher order system is being reduced to a lower order system using the above methods. Then a better approach to reduced order modeling is being suggested.

Organization of the paper is as follows. Section I deals with the introduction of this paper. Mathematical preliminaries of different approaches are dealt in section II. Section III discusses the comparative efficacy of different methods by taking numerical examples and section IV deals with the simulation results and discussion followed by conclusion in the last section.

## 2. MATHEMATICAL PRELIMINARIES

### A. V Krishnamurthy's Approach[11,21]

Consider higher order system of the transfer function 'H(S)' and it is defined as follows

$$H(S) = \frac{b_{11}s^m + b_{21}s^{m-1} + b_{12}s^{m-2} + b_{22}s^{m-3} \ldots\ldots\ldots\ldots..}{a_{11}s^n + a_{21}s^{n-1} + a_{12}s^{n-2} + a_{22}s^{n-3} \ldots\ldots\ldots\ldots\ldots..} \quad (1)$$

Where $n \geq m$

The Routhstability array for numerator and denominator polynomials of H(s) are shown below in tables I and II respectively where $q \leq p \leq n$. Odd coefficients are in first row and even coefficients in second row.

**Table I**

Numerator stability Array

$b_{11}b_{12}b_{13}b_{14}$……………

$b_{21}b_{22}b_{23}b_{24} \ldots\ldots\ldots\ldots$

$b_{31}b_{32}b_{33} \ldots\ldots\ldots\ldots\ldots\ldots..$

$b_{41}b_{42}b_{43} \ldots\ldots\ldots\ldots\ldots\ldots$

$b_{m,1}$

$b_{(m+1),1}$

**Table II**

Denominator Stability Array

$a_{11}a_{12}a_{13}a_{14} \ldots\ldots\ldots\ldots\ldots..$

$a_{21}a_{22}a_{23}a_{24} \ldots\ldots\ldots\ldots$

$a_{31}a_{32}a_{33} \ldots\ldots\ldots\ldots\ldots\ldots..$

$a_{41}a_{42}a_{43} \ldots\ldots\ldots\ldots\ldots\ldots$

$a_{n,1}$

$a_{(n+1),1}$

Generalizing this, the transfer function of a system with reduced order k (<Sn) can simply be constructed with (m+ 2 - k)th and (m + 3 - k)th rows of table I and (n + 1 -k)th and (n + 2- k)th rows of table II.

$$H_K(s) = \frac{b_{(m+2-k),1}s^{k-1} + b_{(m+3-k),1}s^{k-3} + b_{(m+4-k),2}s^{k-3} + \cdots}{a_{(n+1-k),1}s^k + a_{(n+2-k),1}s^{k-1} + b_{(n+3-k),2}s^{k-2} + \cdots}$$

(2)

### B. AGGREGATION BY CONTINUED FRACTION METHOD [20]

Let the Original higher order transfer function

$$H(S) = \frac{a_{21} + a_{22}s + a_{23}s^2 + a_{24}s^3 \ldots\ldots\ldots.a_{2n}s^{n-1}}{a_{11} + a_{12}s + a_{13}s^2 + a_{14}s^3 \ldots\ldots\ldots\ldots a_{2n}s^n}$$

(3)

The original system can be transformed to an aggregated form using a transformation matrix P, corresponding to its continued fraction expansion, i.e.,

$$\frac{dq}{dt} = A_r q + B_r \quad (4)$$

$$v = C_r q \quad (5)$$

Where the $A_r$ = system matrix of order 'r' (r < n) transformed vector q is

$$q = Px \quad (6)$$

& matrix P is obtained through the modified Routh Hurwitz array

| | | | |
|---|---|---|---|
| $a_{11}$ | $a_{12}$ ………. | $a_{1n}$ | 1 |
| $a_{21}$ | $a_{22}$………. | $a_{2n}$ | 0 |
| $a_{31}$ | $a_{32}$ ………… | $a_{3n}$1 | |
| $a_{41}$ | $a_{42}$ ………… | $a_{4n}$ | 0 |
| 1 | | | |

(7)

Subsequent rows are developed by cross multiplication

$$a_{31} = \frac{a_{12}a_{21} - a_{11}a_{22}}{a_{21}}$$

From (7) P can be abstracted

$$P = \begin{bmatrix} a_{31} & a_{31} & \cdots & 1 \\ 0 & a_{51} & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \quad (8)$$

And further steps of algorithm is as follows [21]

$H = PAP^{-1}, L = PB,$
$U = [I_r, 0], F = UH$
$B_r = UL, \quad K = UP$
$C_r = C(K^T)inv(KK^T)$

Equation yields the matrices for the reduced order model. The aggregation matrix "K" becomes accessible as part of the model reduction procedure from equation .This method is very suitable for computational purpose and effective

## 3. NUMERICAL EXAMPLE

### A. V Krishnamurthy's Approach

The system transfer function is:

$$M(S) = \frac{s^3+13s^2+45s+49}{s^4+17s^3+87s^2+177s+106} \qquad (9)$$

Step1: Closed loop characteristics equation is

$$s^4 + 17s^3 + 87s^2 + 177s + 106$$

Step2: Applying routh criterion to above characteristic equation:

Denominator Stability Array

| | | | |
|---|---|---|---|
| $S^4$ | 1 | 87 | 106 |
| $S^3$ | 17 | 177 | |
| $S^2$ | 76.58 | 106 | |
| $S^1$ | 153.46 | | |
| $S^0$ | 106 | | |

Step3: Using Krishnamurthy's approach reduced order closed loop characteristics Equation

$$C_r(s) = 76.58s^2 + 153.48s + 106 \qquad (10)$$

Step 4: Applying routh criterion to numerator:

Numerator Stability Array

| | | |
|---|---|---|
| $S^3$ | 1 | 45 |
| $S^2$ | 13 | 49 |
| $S^1$ | 41.23 | |
| $S^0$ | 49 | |

Step5: Using Krishnamurthy's approach reduced Numerator

$$N_r(s) = 41.23s + 49 \qquad (11)$$

Hence the reduced order transfer function is obtained from equation (17) and (18)

$$G_r(s) = \frac{41.23s+49}{76.58s^2+153.46s+106} \qquad (12)$$

### *B. AGGREGATION BY CONTINUED FRACTION METHOD*

The original system of (9) is of fourth order,

$$M(S) = \frac{s^3 + 13s^2 + 45s + 49}{s^4 + 17s^3 + 87s^2 + 177s + 106}$$

It is reduced second order model is obtained.
Routh array is formed from numerator and denominator of the system

| | | | | |
|---|---|---|---|---|
| 106 | 177 | 87 | 17 | 1 |
| 49 | 45 | 13 | 1 | |
| 79.65 | 58.87 | 14.83 | 1 | |
| 8.78 | 3.87 | 0.38 | | |
| 23.76 | 11.38 | 1 | | |
| -0.33 | 0.01 | | | |
| 12.06 | 1 | | | |
| 0.03 | | | | |
| 1 | | | | |

$$P = \begin{bmatrix} 79.65 & 58.87 & 14.83 & 1 \\ 0 & 23.76 & 11.38 & 1 \\ 0 & 0 & 12.06 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} -1.3308 & -0.7999 & 0.0587 & -0.0981 \\ -1.3308 & -4.1521 & 0.3107 & -0.4478 \\ -1.3308 & -4.1521 & -1.6594 & 2.2024 \\ -1.3308 & -4.1521 & -1.6594 & -9.8576 \end{bmatrix}$$

Matrix U to obtain Second order system can be written as:

$$U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} -1.3308 & -0.7999 & 0.0587 & -0.0981 \\ -1.3308 & -4.1521 & 0.3107 & -0.4478 \end{bmatrix}$$

Second order reduced model is obtained as

$$A_r = \begin{bmatrix} -1.3308 & -0.7999 \\ -1.3308 & -4.1521 \end{bmatrix}$$

$$B_r = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$C_r = [0.6157 \quad 0.3631]$$

$$D_r = 0$$

Hence the reduced order transfer function is:

$$M_r(s) = \frac{0.9788s+2.0641}{s^2+5.4829s+4.4611} \quad (13)$$

## 4. SIMULATION RESULTS AND DISCUSSION

The step response of the original system and the reduced system was plotted in MATLAB and the results were studied. Fig. 1 shows the responses of original model and second order reduced model obtained using V Krishnamurthy's and Aggregation by continued fraction method. The various performance parameters of the second order systems were tabulated in table III.

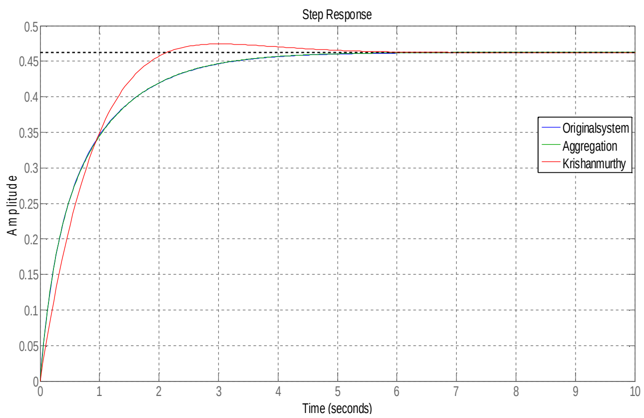


**Fig. 1: Step response of the original system and the reduced system**

**Table III: Qualitative comparison of different systems obtained**

| Graph Name | Rise Time(s) | Settling Time(s) | Peak Value(s) | PeakTime (s) |
|---|---|---|---|---|
| Original system | 1.8749 | 3.5443 | 0.4615 | 6.0290 |
| Aggregation ContinuedFraction | 1.8766 | 3.5472 | 0.4626 | 8.1862 |
| Krishnamurthy's Approach | 1.3542 | 3.7753 | 0.4744 | 3.0335 |

## 5. CONCLUSION

In this paper the realization of a reduced order model substitute to a higher order system based on proposed method of Aggregation has been carried out. Aggregation method harnesses the continued fraction technique. In order to investigate the efficacy of the proposed technique, it is compared with the V Krishnamurthy's approach on reduced order modeling by taking a fourth order system. Subsequently this fourth order system is reduced to second order system using both the methods. The various performance parameters of the reduced systems and the original system obtained from the graphs are compared and analyzed. It is observed that the performance parameters of the proposed method are more close to original system than by V Krishnamurthy's approach.

## REFERENCES


[1] Nise, N.S. Control Systems Engineering. Addison-Wesley, 1995.

[2] Friedland, B. Control System Design - An Introduction to State-Space Methods. McGraw-Hill International Edition, Singapore, 1987.

[3] K. Ogata, "Modern Control Engineering", Prentice Hall, USA, Third Edition 1997.

[4] Robert J. Dudginski and Richard D. Colgren, "Time Domain Effects ofModel Order Reduction," Proceedings IEEE International Symposiumon Intelligent Control, 1989, pp.603-607.

[5] D. Bonvin& D. A. Mellichamp, "A unified derivation and criticalreview of model approaches to model reduction," int. J. Contr., vo1.35, pp. 829-848, 1982.

[6] R.E. Skelton, "Cost decomposition of linear systems with applicationto model reduction," into J. Contr., vol. 32, pp. 1031-1055,1980.

[7] O. Touhami, et al., "Dynamics separation of induction machine modelsusing Gerschgorin circles and singular perturbations,"1st IEEEinternational conference on Electrical and Electronics Engineering, pp. 549-556, 2004.

[8] O. Touhami& M. F. Hamis, "Reduction of the model order ofpermanent magnet synchronous machine using Gerschgorin's circlesand singular perturbations," 38th IEEE Southeasternsymposium onsystem theory, USA, pp. 482-486, 2006.

[9] Y. Shamash," Stable reduced order models usingPade type approximation," IEEE Trans., Automat. Contr., vol. 19, pp. 615-616, 1974.

[10] M. F. Hutton and B. Friedland, "Routhapproximations for reducing order of linear, time invariantsystems," IEEE Trans. Automat. Contr., vol. 20, pp. 329-337, 1975.

[11] V. Krishnamurthy and V. Seshadri, "Modelreduction using the Routh stability criterion,"IEEE Trans. Automat. Contr., vol. 23, pp. 729-730, 1978.

[12] R. Prasad, S. P. Sharma and A. K. Mittal, "Linear model reduction using the advantages ofMikhailov criterion and factor division," Journalof Institution of Engineers, vol. 84, pp. 7-10, 2003.

[13] T. N. Lucas, "Biased model reduction by factordivision," Electronics Letters, vol. 20, p. 582, 1984.

[14] J. Pal, "An algorithm method for thesimplification of linear dynamic systems, "International Journal of Control, vol. 43, pp. 257, 1986.

[15] Y. Shamash, "Truncation method of reduction: Aviable alternative," Electron. Letter, vol. 17, pp.97-99, 1981.

[16] S. Mukherjee and R. N. Mishra, "Order reductionof linear systems using an error minimizationtechnique," J. Frank. Inst., vol. 323, pp.23-32, 1987.

[17] R. Parthasarthy and S. John," Cauer continuedfraction methods for model reduction," Electron. Letter, vol. 17, no. 21, pp. 792-793, 1981.

[18] A. M. Davidson, "Balanced systems and modelreduction," Electron. Letter, vol. 22, pp. 531-532, 1986.

[19] G. D. Howitt and R. Luss, "Model reduction byminimization of integral square errorperformance indices," J. Frank. Inst., vol. 327,pp. 343-357, 1990.

[20] Dr.Ibraheem, Dr. Prabhat Kumarand SadafNaqvi, "Aggregation by Continued Fraction method of model reduction: An application to SISO

system" International Journal of Engineering and Management Research, Vol. 2, Issue-3, JUNE 2012

[21] Chandan Kumar, S. K. Jha and P. Gaur, "Investigations of Model Order Reduction techniques for large scale linear systems", Proceedings of IEEE 5th India International Conference on Power Electronics(IICPE), pp. 1-5, DTU, New Delhi, Dec. 2012.

[22] S.K.Jha, ShivamSinghal, SanchitGupta,"comparative Analysis of different methods of model order reduction for linear, time-invariant systems",IEEE International Conference on Power Electronics, Intelligent Control and Energy Systems (ICPEICES),16673254, July-2016

[23] Pushpendra Kumar Lodhwal and S. K. Jha, "Performance Comparison of Different type of Reduced Order Modeling Methods," Proceedings of the Third International Conference on Advanced Computing & Communication Technologies (ACCT 2013) held 6-7 April 2013 in Rohtak, Haryana, India.